The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, AUTUMN SEMESTER 2020-2021

MECHANICS OF SOLIDS

Time allowed ONE hour plus 30 minutes upload period

Open-book take-home examination

Answer ALL questions

You must submit a single pdf document, produced in accordance with the guidelines provided on take-home examinations, that contains all of the work that you wish to have marked for this open-book examination. Your submission file should be named in the format '[Student ID]_[MMME2053].pdf'.

Write your student ID number at the top of each page of your answers.

This work must be carried out and submitted as described on the Moodle page for this module. All work must be submitted via Moodle by the submission deadline. ***Work submitted after the deadline will not be accepted without a valid EC.***

No academic enquiries will be answered by staff and no amendments to papers will be issued during the examination. If you believe there is a misprint, note it in your submission but answer the question as written.
Contact your Module Teams Channel or SS-AssessEng-UPE@exmail.nottingham.ac.uk for support as indicated in your training.

Plagiarism, false authorship and collusion are serious academic offences *as defined in the University's Academic Misconduct Policy and will be dealt with in accordance with the University's Academic Misconduct Procedures. The work submitted by students must be their own and you must declare that you understand the meaning of academic misconduct and have not engaged in it during the production of your work.*

ADDITIONAL MATERIAL: Formula sheet

All questions carry equal marks

1. Fig. Q1 shows a beam which is simply supported at positions A and E and has an applied point moment, M_C, at position C, and two applied point loads, P_B and P_D, at positions B and D, respectively. Taking the origin as the left-hand side of the beam, which of the following expresses the 2nd order differential equation for this beam?

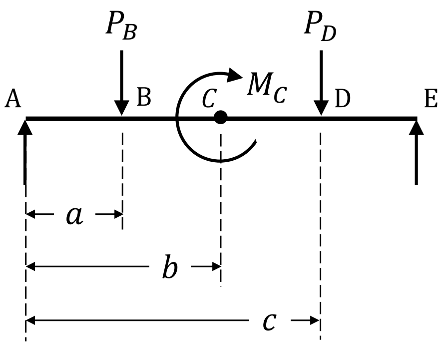

Fig. Q1

A. $EI\frac{d^2y}{dx^2} = R_Ax + M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle$

B. $EI\frac{d^2y}{dx^2} = M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle$

C. $\frac{d^2y}{dx^2} = M_C\langle x-b\rangle - P_B\langle x-a\rangle - P_D\langle x-c\rangle$

D. $EI\frac{d^2y}{dx^2} = R_Ax + M_C\langle x-b\rangle - P_B\langle x-a\rangle - P_D\langle x-c\rangle$

E. $\frac{d^2y}{dx^2} = R_Ax + M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle$

2. What is the corresponding expression for slope in the beam shown in Fig. Q1?

A. $EI\frac{dy}{dx} = M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2}$

B. $\frac{dy}{dx} = \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2} + A$

C. $\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_Ax^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2} + A\right)$

D. $\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_Ax^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2}\right)$

E. $EIy = \frac{R_Ax^3}{6} + \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6}$

3. What is the corresponding expression for deflection in the beam in shown Fig. Q1?

A. $EIy = \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6}$

B. $EI\frac{dy}{dx} = \frac{R_A x^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2}$

C. $y = \frac{M_C\langle x-b\rangle^3}{6} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6} + Ax + B$

D. $y = \frac{1}{EI}\left(\frac{R_A x^3}{6} + \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6}\right)$

E. $y = \frac{1}{EI}\left(\frac{R_A x^3}{6} + \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6} + Ax + B\right)$

4. Fig. Q4 shows the representation of von Mises yield locus at two different hydrostatic stress values. Which stress component from those listed contributes to yielding?

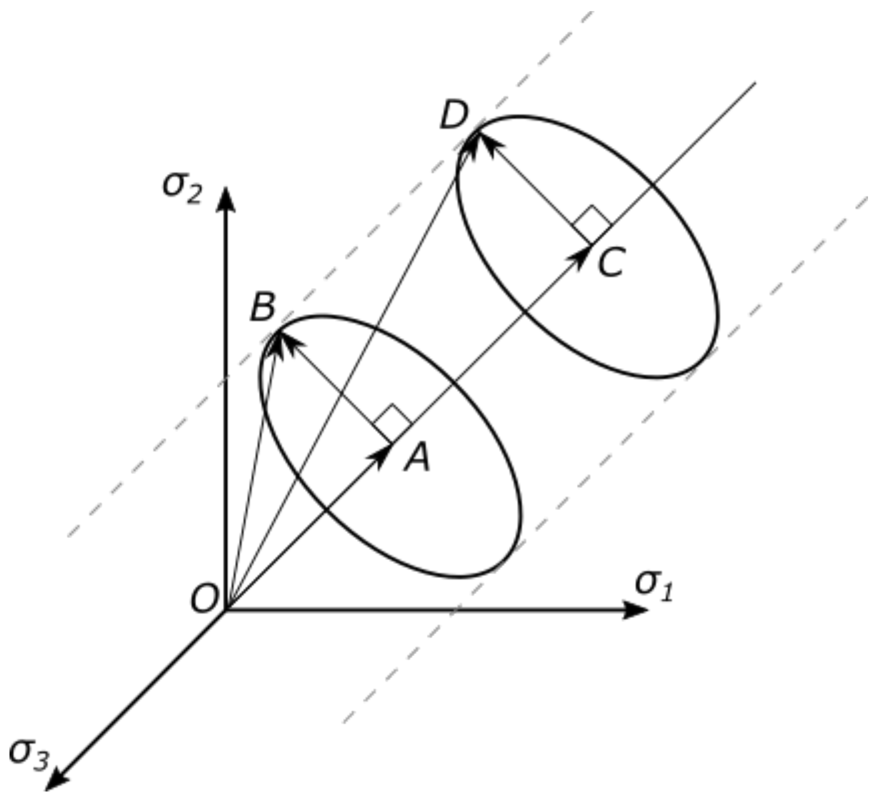

Fig. Q4

A. OA
B. AB
C. OB
D. AC
E. OD

5. A solid bar of $38\ \mathrm{mm}$ diameter is subjected to a torque of $1.1\ \mathrm{kNm}$ and a pure bending moment of $1.3\ \mathrm{kNm}$, according to the von Mises yield criterion; does yielding occur if the uniaxial yield stress of the material is $600\ \mathrm{MPa}$?

A. Yes
B. -
C. -
D. -
E. No

6. If a beam has a rectangular cross-section, as shown in Fig. Q6, has a yield stress, $\sigma_y = 205\ \mathrm{MPa}$, and is subjected to a pure bending moment of $210\ \mathrm{kNm}$, does yielding occur?

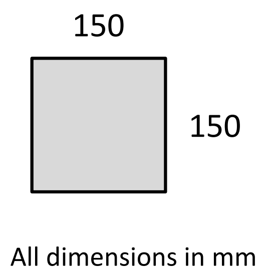

Fig. Q6

Assume elastic-perfectly-plastic material behaviour.

A. Yes
B. -
C. -
D. -
E. No

7. Fig. Q7 illustrates which type of material behaviour?

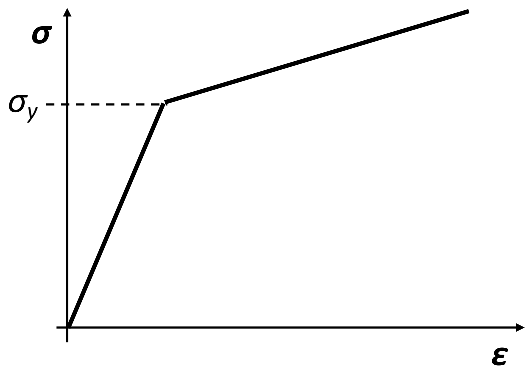

Fig. Q7

- **A.** elastic-perfectly-plastic
- **B.** linear softening
- **C.** non-linear elasticity
- **D.** non-linear hardening
- **E.** linear hardening

8. An internally pressurised cylinder of OD 1 m is subjected to a pressure of 20 bar, what is the smallest value of wall thickness to avoid yielding according to the Tresca yield criterion if the material has a yield strength of 200 MPa?

- **A.** 10 mm
- **B.** 8 mm
- **C.** 6 mm
- **D.** 4 mm
- **E.** 2 mm

9. An unconstrained, 1.9 m long steel bar is subjected to a temperature change of 15 °C, what is the change in length of the bar? ($\alpha = 11 \times 10^{-6} \text{ °}C^{-1}$, $E = 200 \text{ GPa}$)

- **A.** $1.65 \times 10^{-4} \text{ m}$
- **B.** $2.09 \times 10^{-5} \text{ m}$
- **C.** $-3.14 \times 10^{-4} \text{ m}$
- **D.** $3.14 \times 10^{-4} \text{ m}$
- **E.** $-1.65 \times 10^{-4} \text{ m}$

10. What is the limiting pressure to avoid yielding for a cylindrical steel pressure vessel $1.15\ \mathrm{m}$ in diameter and $4\ \mathrm{mm}$ thick according to the von Mises yield criterion and assuming a safety factor of 1.5? ($\sigma_y = 215\ \mathrm{MPa}$)?

A. $0.58\ \mathrm{MPa}$
B. $1.00\ \mathrm{MPa}$
C. $1.15\ \mathrm{MPa}$
D. $1.35\ \mathrm{MPa}$
E. $2.30\ \mathrm{MPa}$

11. At larger mean stresses and when conservatism is required in the prediction of fatigue life:

A. The Gerber curve is a better option than the Soderberg line or the Goodman line
B. The Goodman line is a better option than the Soderberg line or the Gerber curve
C. The Soberberg line is a better option than the Goodman line or the Gerber curve
D. There is no difference in the Soderberg line, the Goodman line or the Gerber curve
E. Either of the Gerber curve or the Goodman line is the best option

12. The Aluminium bar in the compound bar assembly shown in Fig. Q12 is subjected to a temperature change of $5\,°\mathrm{C}$ while the Copper bar has no temperature change. What will the stress in the Copper bar be if both bars have a cross sectional area of $50\ \mathrm{mm}^2$?
($E_{copper} = 128\ \mathrm{GPa}$, $\alpha_{alu} = 23 \times 10^{-6}\ °\mathrm{C}^{-1}$, $E_{alu} = 70\ \mathrm{GPa}$)

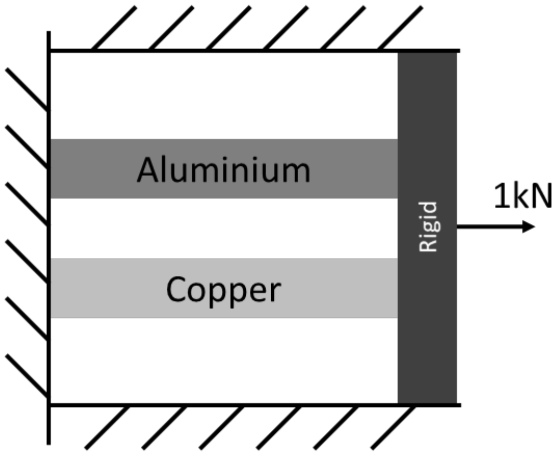

Fig. Q12

A. – 1.9 MPa
B. 7 MPa
C. 13 MPa
D. 1.9 MPa
E. 18 MPa

13. Which of the cross-sections shown in Fig. Q13 has the largest 2nd moment of area about the Y-Y axis (which passes through the centroids of each cross-section)?

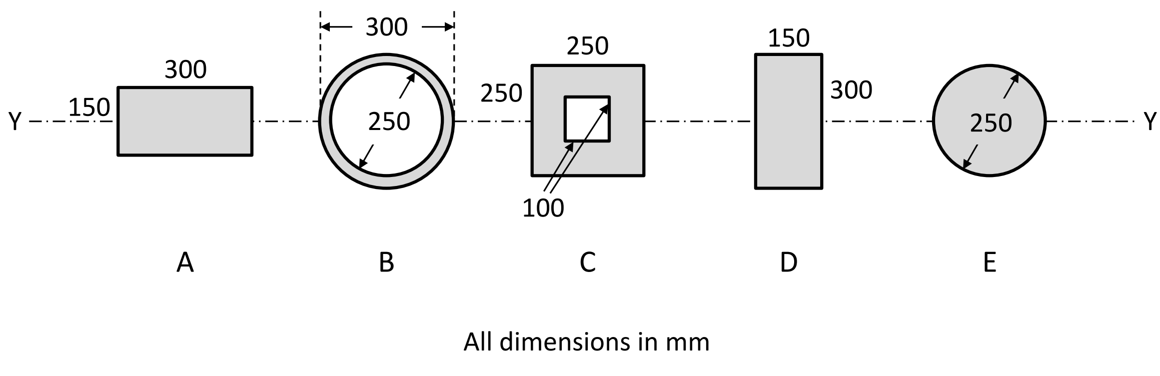

Fig. Q13

A. A
B. B
C. C
D. D
E. E

14. What would the edge length of a square cross-section need to be in order to match the largest 2nd moment of area from Q13?

A. $383.4\ \mathrm{mm}$
B. $252.3\ \mathrm{mm}$
C. $504.6\ \mathrm{mm}$
D. $191.7\ \mathrm{mm}$
E. $248.4\ \mathrm{mm}$

15. Which of the shaft loading cases below results in the largest value of maximum in-plane principal stress acting on a 2D plane-stress element on the surface of a $25\ \mathrm{mm}$ radius, solid shaft? T = Torque, P = Axial Force, M = Pure Bending Moment.

A. $T = 200\ \mathrm{Nm}$, $P = 200\ \mathrm{N}$, $M = 200\ \mathrm{Nm}$
B. $T = 200\ \mathrm{Nm}$, $P = 220\ \mathrm{N}$, $M = 180\ \mathrm{Nm}$
C. $T = 220\ \mathrm{Nm}$, $P = 180\ \mathrm{N}$, $M = 200\ \mathrm{Nm}$
D. $T = 180\ \mathrm{Nm}$, $P = 200\ \mathrm{N}$, $M = 220\ \mathrm{Nm}$
E. $T = 350\ \mathrm{Nm}$, $P = 100\ \mathrm{N}$, $M = 100\ \mathrm{Nm}$

16. What is the value of the maximum in-plane principal stress for the 2D plane-stress element shown in Fig. Q16?

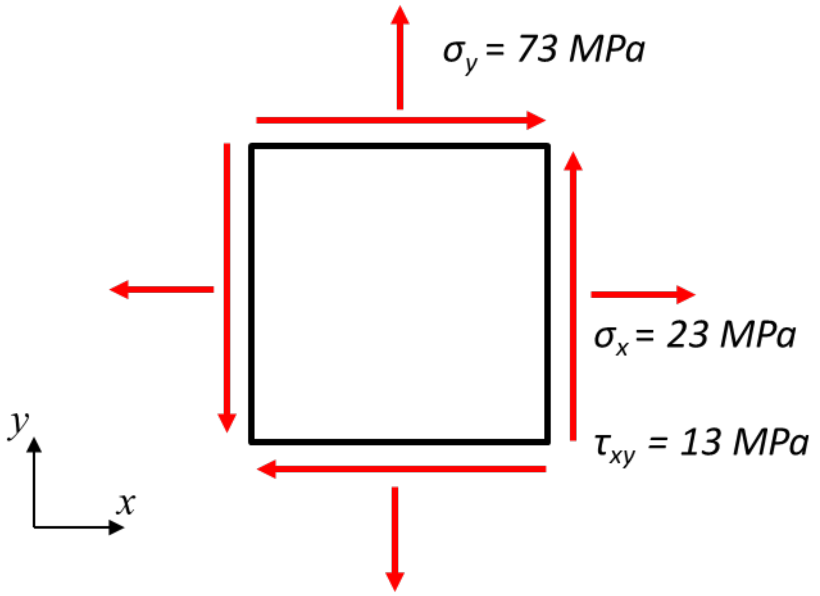

Fig. Q16

A. 31.3 MPa
B. 59.5 MPa
C. 76.2 MPa
D. 87.7 MPa
E. 28.18 MPa

17. For a cracked component, for which $K_I = 2.75\sigma\sqrt{\pi a}$, $K_{Ic} = 120\ \mathrm{MPa}\sqrt{\mathrm{m}}$ and $\sigma_y = 225\ \mathrm{MPa}$, calculate the critical crack length, a_c, if the material behaves in a linear elastic manner and $\sigma = \frac{2}{3}\sigma_y$.

A. 84.63 mm
B. 92.6 mm
C. 2.99 mm
D. 11.97 mm
E. 26.94 mm

18. Fig. Q18 is a plot of crack growth rate, $\frac{da}{dN}$, vs ΔK. The three curves were each produced using the same maximum fatigue load.

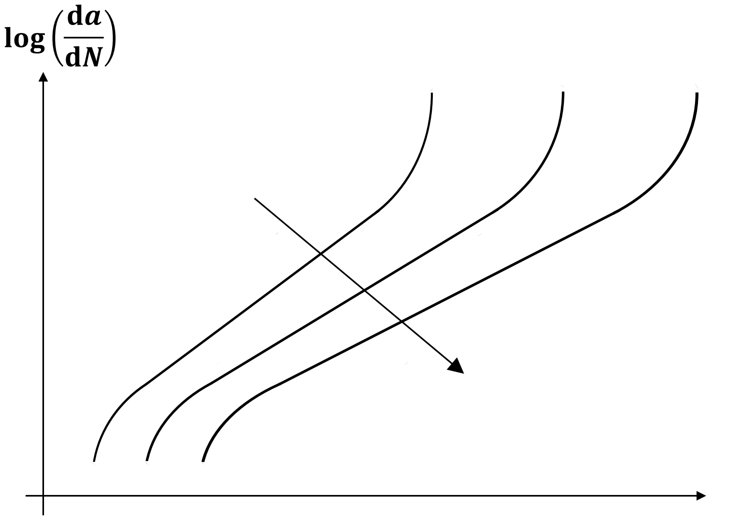

Fig. Q18

The arrow dissecting the curves indicates:

A. Constant mean load
B. Decreasing R-ratio
C. Increasing R-ratio
D. Constant minimum load
E. Constant R-ratio

19. A solid (circular cross-section) shaft will carry a torque of $18\ \text{kNm}$ and will be made of a material with a yield stress of $400\ \text{MPa}$. According to the von Mises yield criteria, what should the diameter be to avoid yielding?

A. $30.6\ \text{mm}$
B. $3.06 \times 10^{-3}\ \text{m}$
C. $24.2\ \text{mm}$
D. $2.42 \times 10^{-3}\ \text{m}$
E. $3.8\ \text{mm}$

20. An aluminium rod with a cross-sectional area of $75\ \text{mm}^2$ is stretched between two fixed points. The tensile load at $20\ °\text{C}$ is $2100\ \text{N}$. What will be the stress at $100\ °\text{C}$? Assume $\alpha = 22 \times 10^{-6}\ °\text{C}^{-1}$, $E = 70\ \text{GPa}$.

A. $-23.5\ \text{MPa}$
B. $-94.8\ \text{MPa}$
C. $123.2\ \text{MPa}$
D. $126\ \text{MPa}$
E. $94.8\ \text{MPa}$

END